An Effective Method for Designing Nonuniformly Coupled Transmission-Line Filters

Gaobiao Xiao, Ken'ichiro Yashiro, *Member, IEEE*, Ning Guan, *Member, IEEE*, and Sumio Ohkawa, *Senior Member, IEEE*

***Abstract*—An effective method is presented in this paper to design nonuniformly coupled transmission-line filters. This kind of filter can be used to realize almost an arbitrary filter characteristics as nonuniform transmission-line filters do. They can be cascaded directly, and no directional coupler is needed. This method begins with calculating characteristic impedance of a nonuniform transmission line by those techniques already established for designing nonuniform transmission-line filters, then determining the even- and odd-mode impedances of a nonuniformly coupled transmission line by a simple transformation. A first-order solution to the related nonlinear equations is especially proposed to calculate effective dielectric constants and line dimensions from the obtained characteristic impedances simultaneously. A device was built and tested, and the measured results verify that the proposed method is of practical use.**

***Index Terms*—Coupled transmission lines, distributed parameter filters, inverse problems.**

I. Introduction

Nonuniform transmission lines (NTLs) have been widely used in filter designs, pulse formation, impedance transducers, and other applications [1]–[8]. As the incident and reflected waves coexist in the same NTL, an additional directional coupler is needed in many cases to separate the two waves, such as in NTL filters. On the other hand, when satisfies some conditions, a nonuniformly coupled transmission line (NCTL) can separate the reflected wave from the incident wave automatically, it seems a good alternative of an NTL in filter designs. However, conventionally, NCTLs have been often used under the following limitations:

1) while the coupling gap between the two lines is nonuniform, each transmission line itself is uniform;
2) coupling is loose;
3) propagating fields are regarded as TEM type, and the even- and odd-mode waves travel in a same speed, thus, the effective dielectric constants corresponding to each mode are chosen to be equal (e.g., $(\epsilon_{\text{eff}}^e + \epsilon_{\text{eff}}^o)/2$, where ϵ_{eff}^e and ϵ_{eff}^o are the effective dielectric constant for the even and the odd modes, respectively).

This paper presents an effective method to avoid these limitations. It also enable us to utilize an NCTL almost as freely as to use an NTL. The proposed design method has the following two properties: 1) both the coupling gap and the width of the transmission lines are nonuniform and 2) the even- and odd-mode waves are no longer regarded as traveling in same speed. A first-order solution is available to solve the construction problem involving ϵ_{eff}^e, ϵ_{eff}^o, which are not equal and also not constant along the lines. An elementary analysis of NCTLs is also reviewed in this paper, which shows that, in filter designs, an NTL can be replaced by an NCTL for some types of coupling transmission-line configurations. However, in the case of coupled-microstrip lines, the condition of $Z_e(y) \geq Z_o(y)$ is a practical limitation for even- and odd-mode characteristic impedances. This means that, for an arbitrary microstrip NTL, we cannot always find a coupled-microstrip line to replace it without distortions. Thus, a proper design procedure is developed, which confines the effect of the replacement to a very low-frequency range, while the frequency characteristics (mainly amplitude and group delay) of the passband that we are concerned about remain almost unaffected.

II. Design Theory

Suppose that $Z^a(x)$ and $Z^b(x)$ are the characteristic impedances of two transmission lines a, b, respectively; their effective dielectric constants are denoted by ϵ_{eff}^a and ϵ_{eff}^b, where x represents the electrical position that is defined by [9]

$$x = \int_0^y \sqrt{L(s)C(s)}ds \tag{1}$$

y denotes the physical position for a transmission line. Usually, it can be further written as

$$x^a = \int_0^{y^a} \frac{1}{c\sqrt{\epsilon_{\text{eff}}^a(s)}}ds, \qquad \text{for line } a \tag{2}$$

$$x^b = \int_0^{y^b} \frac{1}{c\sqrt{\epsilon_{\text{eff}}^a(s)}}ds, \qquad \text{for line } b \tag{3}$$

where c is the light velocity in free space.

Our design method is based on the following important principle. If $Z^a(x)Z^b(x) = Z_0^2$ for $x \in (0, \ell)$, and line a and b are loaded with Z_0 at $x = 0$ and $x = \ell$, we then have

$$S_{11}^a(j\omega) = -S_{11}^b(j\omega) \tag{4}$$

while the incident and reflected waves related to the above S-parameter are defined by

$$a_i^j = \frac{1}{2}\left(\frac{V^j(i)}{\sqrt{Z^j(i)}} + I^j(i)\sqrt{Z^j(i)}\right) \tag{5}$$

Manuscript received April 7, 2000; revised July 19, 2000.

G. Xiao is with the Graduate School of Natural Science, Chiba University, 263-8522 Chiba City, Japan.

K. Yashiro, N. Guan, and S. Ohkawa are with the Faculty of Engineering, Chiba University, 263-8522 Chiba City, Japan.

Publisher Item Identifier S 0018-9480(01)03987-4.

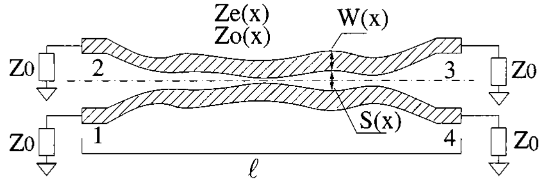


Fig. 1. NCTL.

$$b_i^j = \frac{1}{2}\left(\frac{V^j(i)}{\sqrt{Z^j(i)}} + I^j(i)\sqrt{Z^j(i)}\right) \tag{6}$$

here, $i = 0, \ell$ and $j = a, b$. We have provided one proof of this result in the Appendix.

Next, let us consider a symmetrical NCTL depicted in Fig. 1. The even- and odd-mode characteristic impedances of the NCTL are denoted by $Z_e(x),\ Z_o(x)$, and the effective dielectric constant are denoted by ϵ_{eff}^e and ϵ_{eff}^o, respectively. The overall S-parameters of the NCTL can be written in terms of the even- and odd-mode S-parameters $S^e(j\omega)$ and $S^o(j\omega)$ as

$$S_{11}(j\omega) = \frac{1}{2}\Big[S_{11}^e(j\omega) + S_{11}^o(j\omega)\Big] \tag{7}$$

$$S_{21}(j\omega) = \frac{1}{2}\Big[S_{11}^e(j\omega) - S_{11}^o(j\omega)\Big]. \tag{8}$$

In all these cases, the incident and reflected waves are defined with similar impedance Z_0.

Furthermore, from (4), if we choose $Z_e(x)Z_o(x) = Z_0^2$, then (7) and (8) turn into

$$S_{11}(j\omega) = 0 \tag{9}$$

$$S_{21}(j\omega) = S_{11}^e(j\omega) = -S_{11}^o(j\omega). \tag{10}$$

This means that the reflected wave is output from port 2. There is no reflected wave in port 1, or the input impedance at port 1 is Z_0. Thus, they can be cascaded directly. It is worthwhile to point out that, for an arbitrary NCTL, (9) and (10) are true only under the condition of $Z_e(x)Z_o(x) = Z_0^2$, where x is defined by (1). For TEM approximation, (2) and (3) become linear and identical and then (9) and (10) come to the results proposed by [10] and reviewed by [11].

As is analyzed above, in order to design an NCTL filter, we must first calculate the relating characteristic impedance $Z(x)$ of an NTL from a given frequency characteristics of a filter, either by applying Huang's method or by solving the Zakharov–Shabat (ZS)-type inverse scattering problem [12]–[14], then let $Z_e(x) = Z(x)$ and calculate $Z_o(x)$ from $Z_o(x) = Z_0^2/Z_e(x)$. However, what we have obtained are the impedances in terms of the electrical position x throughout the interval $(0, \ell)$. It is still necessary to calculate from them the linewidth $W(y)$ and gap dimension $S(y)$, while y and x satisfy nonlinear equations (2) and (3) simultaneously. We provide a first-order solution for this problem. Denote $Z_{ey}(y) = Z_e(x(y)),\ y \in (0, L_e)$, and $Z_{oy}(y) = Z_o(x(y)),\ y \in (0, L_o)$, where

$$\int_0^{L_e} \frac{1}{c\sqrt{\epsilon_{\text{eff}}^e(s)}} ds = \ell \tag{11}$$

$$\int_0^{L_o} \frac{1}{c\sqrt{\epsilon_{\text{eff}}^e(s)}} ds = \ell. \tag{12}$$

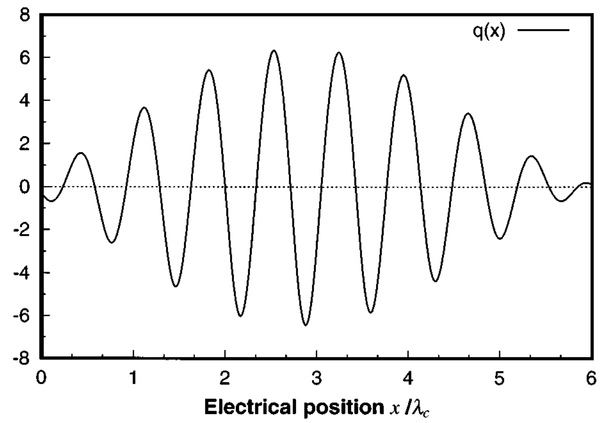


Fig. 2. Constructed potential function $q_{(x)}$.

Usually, $L_e \neq L_o$ because $\epsilon_{\text{eff}}^e(y)$ and $\epsilon_{\text{eff}}^o(y)$ are always not equal. Obviously, this is unsuitable for fabrication. However, we can simply add a uniform line section with impedance Z_0 to the end of the shorter one. No influence on $S_{11}(j\omega)$ and $S_{21}(j\omega)$ will be introduced because each line is required to be matched with Z_0 at its ends. In the case of coupled-microstrip lines, $\epsilon_{\text{eff}}^e \geq \epsilon_{\text{eff}}^o$, thus, $L_e \leq L_o$. We choose the length of the NCTL as L_o, and assume that $Z_{ey}(y) = Z_0$ for $L_e < y \leq L_o$.

Let $y_i = i\Delta y,\ i = 0, 1, 2, \ldots, N$, and $y_N = \max\{L_e, L_o\}$, where N will be automatically determined in the following algorithm. Replace a, b by e, o in (2) and (3) and rewrite them in their discretized form

$$x_{j+1}^e - x_j^e = \frac{1}{c\sqrt{\epsilon_{\text{eff}}^e(y_j)}}(y_{j+1} - y_j) = \frac{\Delta y}{c\sqrt{\epsilon_{\text{eff}}^e(y_j)}} \tag{13}$$

$$x_{j+1}^o - x_j^o = \frac{1}{c\sqrt{\epsilon_{\text{eff}}^o(y_j)}}(y_{j+1} - y_j) = \frac{\Delta y}{c\sqrt{\epsilon_{\text{eff}}^o(y_j)}}. \tag{14}$$

We will determine $W(y)$ and $S(y)$, together with $\epsilon_{\text{eff}}^e(y)$ and $\epsilon_{\text{eff}}^o(y)$ at y_i from $Z_{ey}(y_i)$ and $Z_{oy}(y_i)$ with the following algorithm.

- Step 1) Initially, let $j = 0$, and $y_0 = x_0^e = x_0^o = 0$.
- Step 2) Find $Z_{ey}(y_j) = Z_e(x_j^e)$ and $Z_{oy}(y_j) = Z_o(x_j^o)$. From them, determine $W(y_j)$, $S(y_j)$, $\epsilon_{\text{eff}}^e(y_j)$, and $\epsilon_{\text{eff}}^o(y_j)$ by those formulas in [15] and [16].
- Step 3) Compute x_{j+1}^e and x_{j+1}^o from (13) and (14).
- Step 4) Let $j = j+1$, and repeat steps 2 and 3 until $x_{j+1}^e \geq \ell$ and $x_{j+1}^o \geq \ell$.

III. Example for Microstrip NCTL Filter

The designed filter is expected to have a linear phase, and a passband of $0.6 \leq \omega/\omega_c \leq 0.8$, ω_c is normalizing angular frequency. We also assumed -1.5-dB loss throughout the passband beforehand just because we cannot build a gap narrower than 0.2 mm accurately enough with a circuit-board plotter.

Fig. 2 shows the constructed potential function $q(x)$ for the NTL that is used to realize this filter. It was obtained by solving a ZS-type inverse-scattering problem [12]. Actually, we also adopted Huang's method to calculate $q(x)$ from the local reflection coefficient, and no significant discrepancies occur. Fig. 3 shows the required $Z_e(x)$ and $Z_o(x)$ for an NCTL fabrication. However, the impedances shown in Fig. 3 cannot be realized by

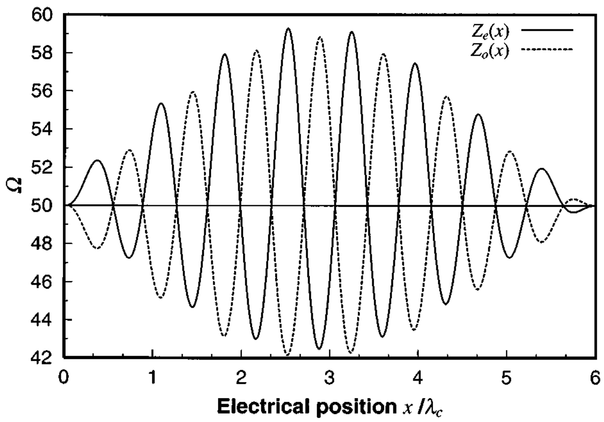


Fig. 3. Even- and odd-mode characteristic impedance for an NCTL.

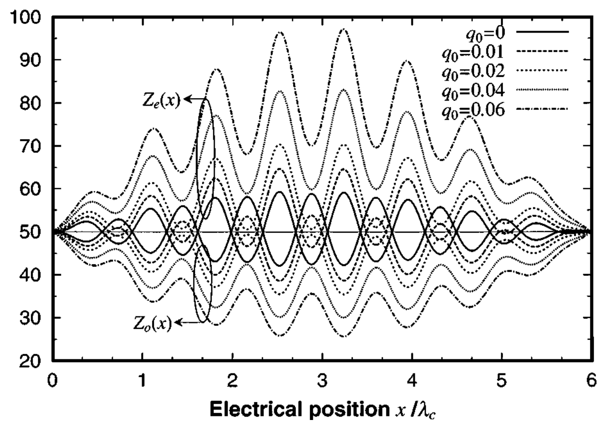


Fig. 4. Effect on $Z_e(x)$ and $Z_o(x)$ by perturbing $q_{(x)}$.

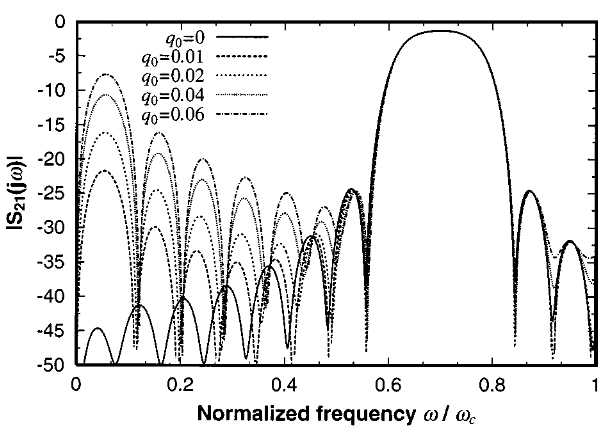


Fig. 5. Effect on $S_{21}(j\omega)$ by perturbing $q(x)$.

a microstrip NCTL because the condition of $Z_e(x) \geq Z_o(x)$ is not satisfied. In this paper, we propose a way to generate proper $Z_e(x)$ and $Z_o(x)$ for a microstrip NCTL through perturbing $q(x)$ by adding a small q_0 to it in $x \in (0, \ell)$. Fig. 4 shows that $Z_e(x)$ and $Z_o(x)$ gradually depart from each other with increasing q_0, and the above condition can be reached when q_0 is sufficiently large. Fig. 5 shows the simulated results of $S_{21}(j\omega)$. It is obvious that the perturbation affects only the characteristics in a lower frequency range, as is expected. It is also true with regard to group delay. We have chosen $Z_e(x)$ and $Z_o(x)$ for our test device, as illustrated in Fig. 6, where $q_0 = 0.017$. In order to

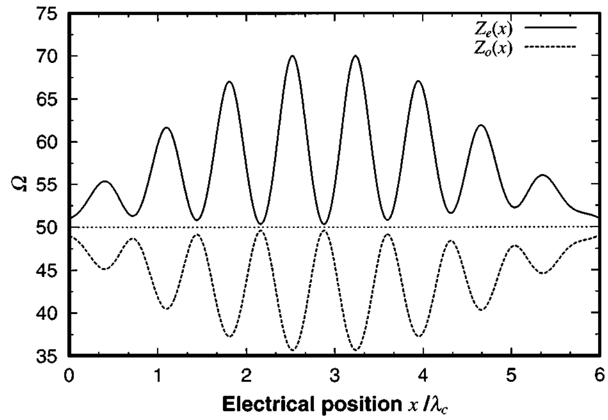


Fig. 6. Selected $Z_e(x)$ and $Z_o(x)$ for the test device.

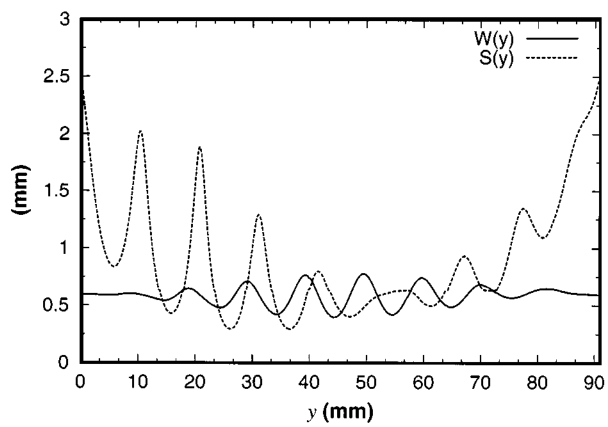


Fig. 7. Device dimensions: transmission-line width $W(y)$ and gap width $S(y)$.

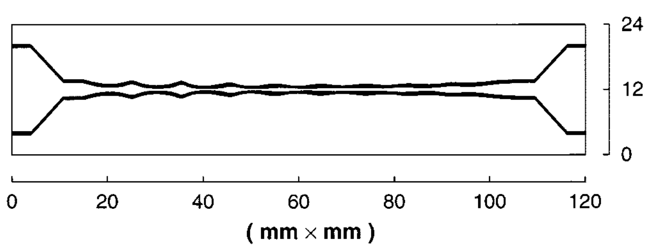
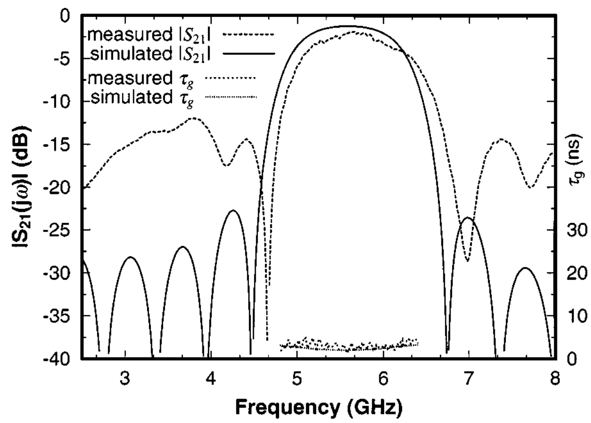
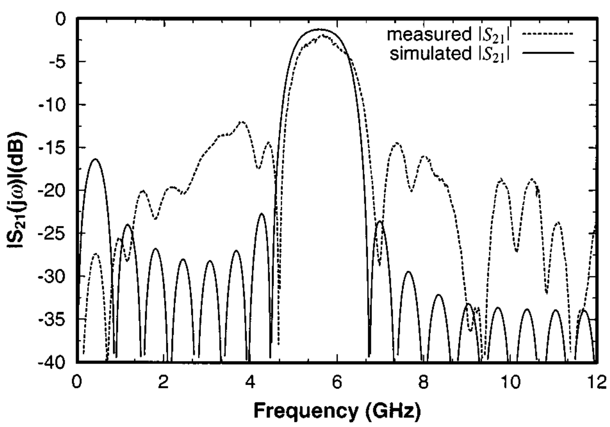


Fig. 8. Pattern for the test device: built on RT/Duroid 6010.5, milled through LPKF ProtoMat 91s/VS.

avoid excessively rapid changes of the gap width near the ends of the NCTL, we selected $Z_e(0) = Z_e(\ell) = 51\ \Omega$, slightly greater than $Z_0 = 50\ \Omega$. Fig. 7 shows the resultant $W(y)$ and $S(y)$, while Fig. 8 shows the pattern of the device. The normalizing frequency is 8 GHz, and the effective dielectric constants are calculated at the central frequency of the passband. The device was built on RT/Duroid 6010.5 printed circuit board with a copper conductor ($\varepsilon_r = 10.2$ and thickness $H = 0.635$ mm) utilizing a circuit-board plotter (LPKF ProtoMat 91s/VS), as we only intend to demonstrate the principle. Short bent line sections are added at the four ends of the NCTL only for the convenience of measurement. Fig. 9(a) and (b) shows the measured results of $S_{21}(j\omega)$. The milling errors affect not only the coupling degree of the gap, but also the effective dielectric constants. The former causes the main error in the amplitude response of the passband,



(a)



(b)

Fig. 9. (a) Measured results for the test device: 2.5–8 GHz, measured through an HP8720C network analyzer. (b) Measured results for the test device: 0.5–12 GHz.

while the latter introduces a shift of the passband. The poor stopband characteristics is mainly due to the mismatch at the ends of the NCTL and the effects of the additional line sections.

IV. Conclusions

With the method presented in this paper, an NCTL can be used almost as widely as an NTL in filter designs. NCTL filters enjoy the advantages that they can be cascaded directly, and no directional coupler is needed. On the other hand, the dispersion effect is not considered in this paper's method, and the shape distortions in the coupling structure are not accounted for while adopting Kirschning and Jansen's formulas.

Appendix

Let $V(y)$ and $I(y)$ be the voltage and current of an NTL with impedance $Z(y)$, then the reflection coefficient $\Gamma(y)$ obeys the following Riccati equation [18]:

$$\frac{d\Gamma(y)}{dy} - 2j\beta(y)\Gamma(y) + \Big(1 - \Gamma^2(y)\Big)k(y) = 0 \qquad (15)$$

where

$$\Gamma(y) = \frac{V(y) - Z(y)I(y)}{V(y) + Z(y)I(y)} \qquad (16)$$

$$k(y) = \frac{1}{2Z(y)}\frac{dZ(y)}{dy}. \qquad (17)$$

When y is replaced by electrical position x, (15) becomes

$$\frac{d\Gamma(x)}{dx} - 2j\beta_0\Gamma(x) + \Big(1 - \Gamma^2(x)\Big)k(x) = 0 \qquad (18)$$

$\beta_0 = \omega/c$. Clearly, for lines a and b with impedance $Z^a(x)$ and $Z^b(x)$ satisfying $Z^a(x)Z^b(x) = Z_0^2$, we have

$$k^a(x) = -k^b(x) \qquad (19)$$

If $\Gamma^a(\ell) = -\Gamma^b(\ell)$, then $\Gamma^a(0) = -\Gamma^b(0)$ must also hold. When the NTL is loaded by Z_0

$$\Gamma^a(\ell) = \frac{Z_0 - Z^a(\ell)}{Z_0 + Z^a(\ell)} \qquad (20)$$

$$\Gamma^b(\ell) = \frac{Z_0 - Z^b(\ell)}{Z_0 + Z^b(\ell)} = -\Gamma^a(\ell) \qquad (21)$$

and

$$\Gamma_g^a = \frac{Z^a(0) - Z_0}{Z^a(0) + Z_0} \qquad (22)$$

$$\Gamma_g^b = \frac{Z^b(0) - Z_0}{Z^b(0) + Z_0} = -\Gamma_g^a \qquad (23)$$

also, from the definition of the S-parameter

$$S_{11}^a(j\omega) = \frac{\Gamma_g^a + \Gamma^a(0)}{1 + \Gamma_g^a\Gamma^a(0)} \qquad (24)$$

$$S_{11}^b(j\omega) = \frac{\Gamma_g^b + \Gamma^b(0)}{1 + \Gamma_g^b\Gamma^b(0)}. \qquad (25)$$

Thus, we have

$$S_{11}^a(j\omega) = -S_{11}^b(j\omega). \qquad (26)$$

References

[1] F. Huang, "Quasitransversal synthesis of microwave chirped filter," *Electron. Lett.*, vol. 28, pp. 1062–1064, May 1992.

[2] H. Cheung, "Low loss quasitransversal microwave filters with specified amplitude and phase characteristics," *Proc. Inst. Elect. Eng.*, pt. H, vol. 140, pp. 433–438, Dec. 1993.

[3] F. Huang *et al.*, "A superconducting microwave linear phase delay line filter," *IEEE Trans. Appl. Superconduct.*, vol. 3, pp. 2778–2781, Mar. 1993.

[4] H. C. H. Cheung, F. Huang, and M. J. Lancaster, "Improvements in superconducting linear phase microwave delay line bandpass filters," *IEEE Trans. Appl. Superconduct.*, vol. 5, pp. 2675–2677, June 1995.

[5] F. Huang, "Frequency dependent transmission line loss in quasitransversal microwave filters," *Proc. Inst. Elect. Eng.*, vol. 141, pp. 402–407, Oct. 1994.

[6] S. C. Burkhart and R. B. Wilcox, "Arbitrary pulse synthesis via nonuniform transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 1514–1518, Oct. 1990.

[7] M. L. Roy, A. Pèrennec, S. Toutain, and L. C. Calvez, "The continuously varying transmission-line technique- application to filter design," *IEEE Trans. Microwave Theory Tech.*, vol. 47, pp. 1680–1687, Sept. 1999.

[8] J. P. Mahon and R. S. Elliott, "Tapered transmission lines with a controlled ripple response," *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 1415–1420, Oct. 1990.

[9] M. W. Wohlers, "A realizability theory for smooth lossless transmission lines," *IEEE Trans. Circuit Theory*, vol. CT-13, pp. 356–363, Sept. 1966.

[10] C. B. Sharpe, "An equivalence principle for nonuniform transmission line directional couplers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-15, pp. 35–37, July 1967.

[11] L. A. Hayden and V. K. Tripathi, "Nonuniformly coupled microstrip transversal filters for analog signal processing," *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 47–53, Jan. 1991.

[12] G. H. Song and S. Y. Shin, "Design of corrugated waveguide filters by the Gelfand–Levitan–Marchenko inverse scattering method," *J. Opt. Soc. Amer. A, Opt. Image Sci.*, vol. 2, no. 11, pp. 1905–1915, 1985.

[13] P. P. Roberts and G. E. Town, "Design of microwave filters by inverse scattering," *IEEE Trans. Microwave Theory Tech.*, vol. 43, pp. 739–743, Apr. 1995.

[14] W. Rundell and P. E. Sacks, "Construction techniques for classical inverse Sturm–Liouville problems," *Math. Comput.*, vol. 58, pp. 161–183, Jan. 1992.

[15] E. Hammerstad and O. Jensen, "Accurate models for microstrip computer-aided design," in *IEEE MTT-S Int. Microwave Symp. Dig.*, 1980, pp. 407–409.

[16] M. Kirschning and R. Jansen, "Accurate wide-range design equations for the frequency-dependent characteristic of parallel coupled microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 83–90, Jan. 1984.

[17] S. Rosloniec, "Design of coupled microstrip lines by optimization methods," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 1072–1074, Nov. 1987.

[18] R. E. Collin, *Foundations for Microwave Engineering*. New York: McGraw-Hill, 1966.

Gaobiao Xiao was born in China, in 1965. He received the M.S. degree from the Huazhong University of Science and Technology, Wuhan, China, in 1988, the B.S. degree from the Changsha Institute of Technology, Changsha, China, in 1991, and is currently working toward the D.E. degree in electrical and electronics engineering at Chiba University, Chiba, Japan.

From 1991 to 1997, he was a Research Associate with the Department of Electrical and Electronics Engineering, Hunan University. His main research interests are in microwave filters, inverse scattering problems, circuits analysis, and numerical methods in the electromagnetic field.

Ken'ichiro Yashiro (M'83) received the B.E. and M.E. degrees in electronics engineering from Chiba University, Chiba, Japan, in 1974 and 1976, respectively, and the D.E. degree from the Tokyo Institute of Technology, Tokyo, Japan, in 1979.

In 1979, he joined the Department of Electrical and Electronics Engineering, Chiba University, as a Research Associate, and became an Associate Professor in 1987 and a Full Professor in 1998. His research interests include magnetostatic wave devices, magnetic solitons, electromagnetic wave scattering, and inverse-scattering problems.

Ning Guan (M'93) received the B.S., M.S., and Ph.D. degrees in electrical and electronics engineering from Chiba University, Chiba, Japan, in 1985, 1987. and 1990, respectively.

Since 1991, he has been with the Department of Electrical and Electronics Engineering, Chiba University, where he is currently a Research Associate. His research interests are in magnetostatic wave devices, electromagnetic waves, and applications of wavelet to boundary value problems in electromagnetic theory.

Sumio Ohkawa (M'82–SM'84) received the B.Eng. degree in electrical engineering from Chiba University, Chiba, Japan, in 1961, and the D.Eng. degree from the Tokyo Institute of Technology, Tokyo, Japan, in 1974.

In 1970, he joined the Department of Electrical Engineering, Chiba University, as a Research Associate, and became an Associate Professor in 1970 and a Full Professor in 1978. His research has been mainly concerned with measurements of materials in the microwave range, applications of magnetostatic wave devices, and magnetostatic wave envelope solitons.